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Stepped Spillway Downstream of a Piano Key Weir – Critical Length for Uniform Flow

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ABSTRACT: Recent researches showed the rapid aeration of the flow downstream of piano key weirs. This could be related to the specific geometric features of the structure, creating several interacting flows and jets downstream. In order to validate this assumption, an experimental research has been carried out at the Laboratory of Engineering Hydraulics, University of Liège. The main objective of the study was to analyze the energy dissipation occurring in skimming flow conditions on a stepped spillway downstream of a piano key weir (PKW). Comparison with energy dissipation on a stepped spillway downstream of a standard ogee crested weir has been performed. An indirect method has been used to determine the residual energy at the spillway toe. The results show that uniform flow conditions are reached faster on a stepped spillway downstream of a PWK compared to the length needed downstream of a standard ogee crested weir. Extrapolation of the results to prototype flow conditions is also discussed with regards to scale effects influence.

Keywords: skimming flow, uniform flow, hydraulic jump, conjugate depths equation, local head loss

1 INTRODUCTION

Stepped spillways are used in dam construction to reduce the length or eliminate the stilling basin at the dam toe by dissipating energy along the spillway. They enable also to decrease the erosion risk due to cavitation by providing natural aeration. They are usually built downstream of standard ogee-crested weirs.

Two distinct flow regimes may occur on stepped spillways: nappe flow and skimming flow. In nappe flow the steps act as a series of overfalls with the water plunging from one step to another. In skimming flow, the water flows as a coherent stream over the pseudobottom formed by the outer step edges, without air pockets under the jets and with development of intense recirculating zones in the triangular cavities formed by the step faces and the pseudobottom. These vortices are maintained by the transmission of shear stress from the coherent stream over the pseudobottom and contribute significantly to the energy dissipation for this type of regime. The nappe flow is found for large steps and low discharge whereas the skimming flow appears for small steps and high discharge. Between these both flow regimes, there is a transition regime where both nappe and skimming flows occur simultaneously in different parts of the stepped spillway (Boes and Hager, 2003a).

For a stepped spillway with skimming flow regime, as studied in this paper, the free-surface of the flow is smooth in the early steps and no air entrainment occurs. Close to the bottom, turbulence is generated. When the outer edge of the turbulent boundary layer reaches the free surface, air entrainment occurs. Downstream of the point of inception of air entrainment, the flow becomes rapidly aerated. A mixture of air and water extends gradually through the fluid. Far downstream, the flow becomes uniform. From this moment, air concentration flow, velocity and water depth remain nearly constant along the spillway (Chanson, 1994). A criterion has been proposed by Boes et Hager (2003a) to predict the minimum vertical distance $H_{dam,u}$ from spillway crest needed to reach uniform flow depending on the critical depth h_c and the slope ϕ of the spillway:

$$\frac{H_{dam,u}}{h_c} = 24\left(\sin\phi\right)^{2/3}. \qquad (1)$$

When uniform flow conditions are reached on a stepped spillway, Boes and Hager (2003a) suggest to calculate the residual energy head $E_{f,u}$ at the spillway end as

$$\frac{E_{f,u}}{E_{\max}} = \frac{F}{\frac{H_{dam}}{h_c} + \frac{3}{2}}, \qquad (2)$$

where E_{max} is maximum reservoir energy head, H_{dam} is the dam height and F is a parameter related to the the bottom roughness. This relation has been established analytically from the general equation for water curves (Hager and Boes, 2000). Its validity has already been proven in several searches by comparison with experimental data.

Equation (2) is independent of the type of weir upstream of the stepped spillway as uniform flow characteristics are related to the discharge and the spillway features.

The specific geometric features of the piano key weirs create several interacting flows and jets downstream of the structure, enhancing thus the aeration of the flow. An experimental study has been conducted at the Laboratory of Engineering Hydraulics, University of Liège, to compare, in an idealized environment, the energy dissipation on a stepped spillway downstream a PKW and downstream of a standard ogee-crested weir. In particular, the study aims at verifying whether uniform flow conditions can be reached faster on stepped spillway placed downstream of the PKW than downstream of an ogee-crested weir.

First results were published by Erpicum et *al.* (2011). Two geometries of PKW were considered as well as a standard ogee-crested weir upstream of the same stepped spillway model. Thanks to sufficient length of the stepped spillway (H_{dam}/h_c=16.1 to 58.8), uniform flow conditions were reached at the spillway toe for almost the whole range of tested discharges. Therefore, the resulting energy at the spillway toe was equivalent downstream of the PKWs and downstream of the standard ogee crested weir. However, for the same specific discharge on the spillway, important differences in the flow features were observed. Indeed, with both PKW geometries, the flow was fully aerated immediately downstream of the PKW, while this was not the case downstream of the ogee-crested weir.

In the present study, the experimental facility used by Erpicum *et al.* (2011) has been modified to reduce the spillway length. Several spillway lengths have been tested to generalize criterion proposed by Boes and Hager (2003a) for stepped spillways downstream of PKWs.

Extrapolation of the results to prototype flow conditions is also discussed. Though not experimentally verified, scale effects are not anticipated due to criteria involving the Weber (Kobus, 1984) and Reynolds (Rutschmann, 1988 and Speerli, 1999) numbers.

2 EXPERIMENTAL FACILITY AND METHODOLOGY

Experimental data were obtained on a facility (Figure 1) made of a 0.494m wide stepped spillway with a slope of 52° and regular steps of 2.4cm in length and 3cm in height. The spillway is linked to an upstream reservoir and a 4.20m long downstream horizontal channel. Thanks to an adoptive support system, the length of the spillway can be easily modified from 0 to 1.30m. All the walls of the facility (sidewalls, channel bottom, weirs and spillway) are made of steel, PVC or Plexiglas in order to minimize friction effects. The upstream reservoir is fed by a regulated pump connected to a pressurized pipes network. The downstream extremity of the horizontal channel is equipped with an adjustable vertical gate.

Tested discharges ranged from 5l/s to 80l/s. They have been measured in the upstream pipe using an electromagnetic flow meter (accuracy of 1%). The water depths in the upstream reservoir and along the horizontal channel have been measured by ultrasonic probes (frequency of 10Hz, resolution of 0.025mm and accuracy of 2%). Because of small free surface oscillations, water depth measurements have been averaged on a period of 90s to define a single water depth value in each point.

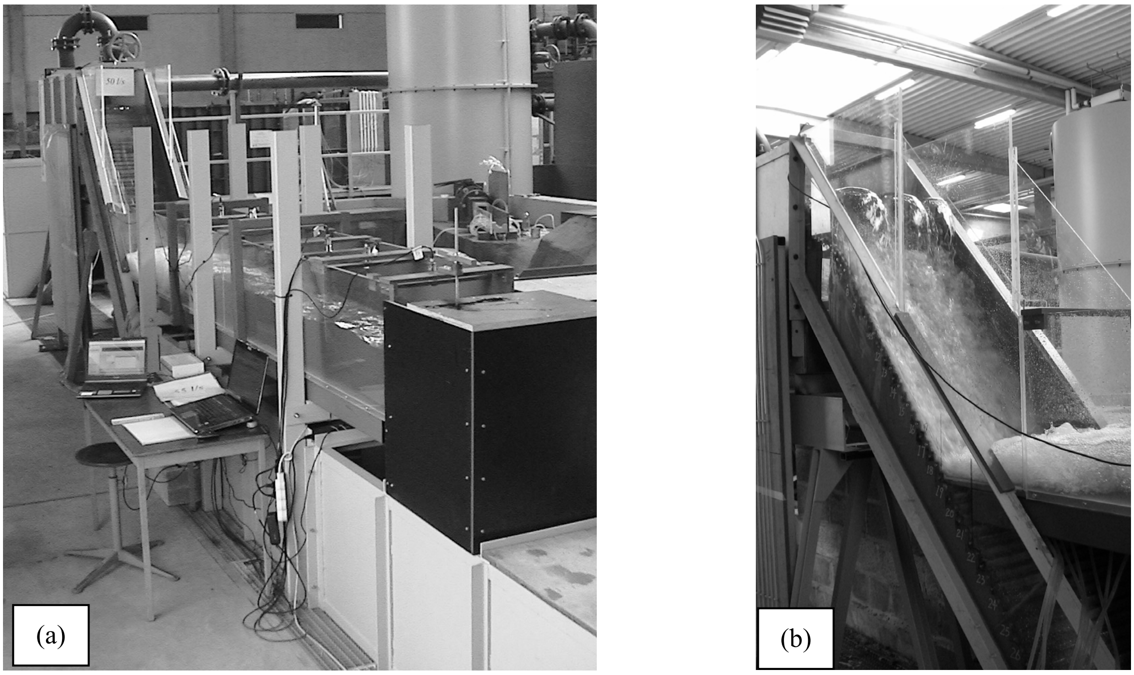

Figure 1: Global view of the experimental facility (a) and view of PKW_2 and spillway [Position II] (b).

In the present study, three different dam heights have been considered, namely Position I, Position II and Position III, corresponding to a spillway length of 1.27m, 1.04m and 0.62m respectively. Three different models of weir have been tested upstream of the stepped spillway: a standard ogee-crested weir (Dewals *et al*., 2004) and two geometries of piano key weirs, noted PKW_1 and PKW_2 (Erpicum *et al*., 2011). The dimensions of the PKW_2 are generally 1.6 times smaller than PKW_1 ones, except the spillway width and the walls thickness which are constant. PKW_1 represents 1.5 inlets and 1.5 outlets while PKW_2 represents 2.5 inlets and 2.5 outlets. Their dimensions are summarized in Table 1. They are both "modified type A PKWs", i.e. with downstream and upstream overhangs of different lengths. Steps have been built in the outlets to improve energy dissipation. The dam's heights (from crest of weir to spillway toe) are given in Table 2 for all combinations of weir's type and Position (I, II or III).

Table 1. General dimensions and aspect ratios of PKW_1 and PKW_2.

	W_i [cm]	W_0 [cm]	T_s [mm]	P [cm]	B [cm]	B_b [cm]	B_i [cm]	B_0 [cm]	W_i/W_0 [-]	P/W_i [-]	T_s/W_i [-]	L_u/W_u [-]
PKW_1	16.9	12.3	15	26.2	62.3	37.4	11	13.9	1.37	1.55	0.09	4.78
PKW_2	9.8	7.7	10	16.3	38.8	23.3	6.8	8.7	1.27	1.66	0.10	4.88

Table 2. Dam heights (H_{dam}) for all tested combinations of weir type and Position (I, II or III).

	Standard weir	PKW_1	PKW_2
$H_{dam,I}$	1.067	1.282	1.183
$H_{dam,II}$	0.887	1.102	1.003
$H_{dam,III}$	0.552	0.767	0.668

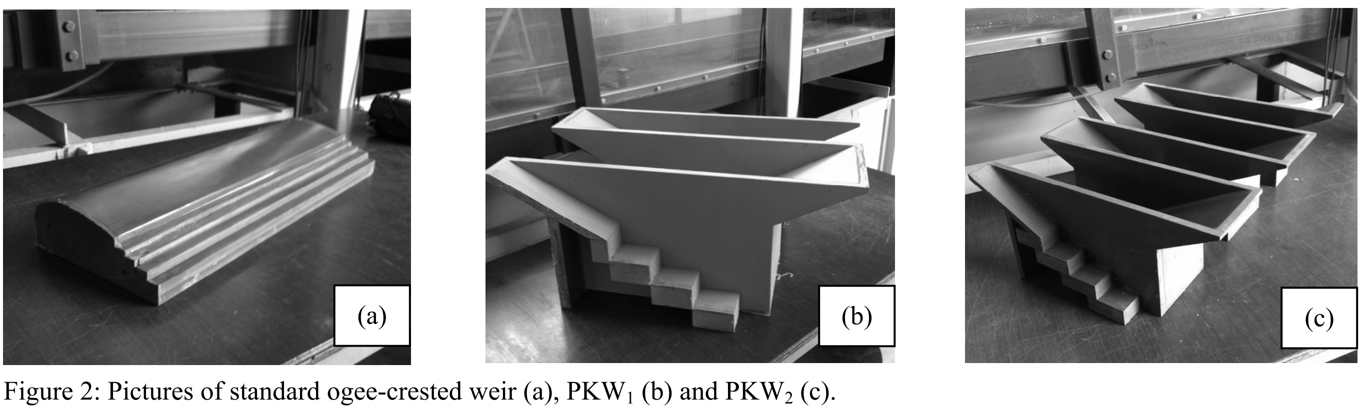

Figure 2: Pictures of standard ogee-crested weir (a), PKW_1 (b) and PKW_2 (c).

Experiments consisted in measuring and comparing, for constant discharges, the energy dissipation along the stepped spillway depending on the type of upstream weir. This energy dissipation has been calculated as the difference between the energy measured in the upstream reservoir E_0 and the energy at the spillway toe E_p.

Because of air entrainment and high flow velocity at the spillway toe, the residual energy E_p is difficult to determine accurately by a direct measurement of the water depth and flow velocity. Therefore an indirect method has been used; consistently with Matos and Quintela (2004), Shvainshtein (1999) or Erpicum *et al*. (2011). It consists in creating a hydraulic jump in the downstream horizontal channel by means of a gate, to measure the flow energy downstream of the jump, where flow depth varies less and aeration rate is smaller, and then to calculate the water depth at the spillway toe and the residual energy using the conjugate depths equation.

Tested discharge ranged from 5l/s to 80l/s (specific discharge from 0.02l/s to 0.162l/s). A sketch of the experimental facility is shown in Figure 4.

3 RESULTS AND DISCUSSIONS

3.1 *Qualitative observations*

The first result of the tests consist in a visual confirmation of previous observations by other researchers such as Erpicum *et al*. (2011) and Ho Ta Khanh *et al*. (2011). For the same specific discharge, important differences have been observed in the flow downstream of the spillway depending on the type of weir (Figure 3).

With both PKW geometries, the flow is fully aerated directly after the weir toe, while this is not the case with the ogee-crested weir. The inception point downstream of a PKW is significantly closer to the spillway crest than downstream of an ogee-crested weir. As a result, the energy dissipation on a stepped spillway downstream of a PKW should be different from downstream an ogee-crested weir. In the next subsection, we show that uniform flow conditions are reached faster downstream of a PKW than downstream of the ogee-crested weir. Another consequence of this higher aeration is a reduction of the cavitation risk, so that it should be possible to avoid the construction of aerators in the non-aerated region.

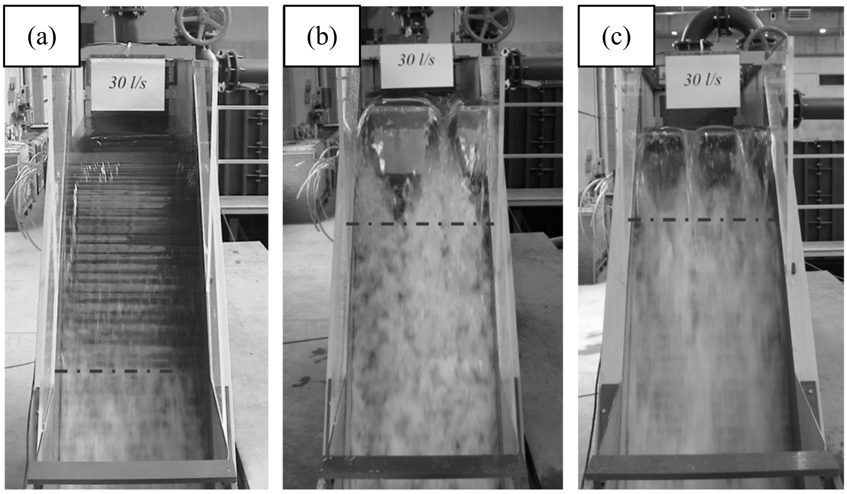

Figure 3: Change in the inception point location along the spillway between standard ogee-crested weir and PKWs (Position I, Q = 30 l/s, q = 0.06 m²/s): standard ogee-crested weir (a), PKW_1 (b) and PKW_2 (c).

3.2 *Quantitative analysis*

For a given discharge, upstream energy E_0 can be evaluated directly from the water depth h_0 measured by the ultrasonic probe in the upstream reservoir. Indeed, under the assumption of a uniform velocity distribution across the reservoir cross-section, E_0 is given by

$$E_0 = z + h_0 + \frac{v^2}{2g} \tag{3}$$

where z is the reservoir bottom elevation from the downstream horizontal channel bottom level and v the mean velocity through the reservoir cross section. To be consistent with the uniform velocity assumption, the probe used to measure the water depth h_0 is located at a horizontal distance from the weir crest longer than twice the maximum head on the weir (Paternoster, 1963).

In all tests, the hydraulic jump was located exactly at the spillway toe (Figure 4 - b).

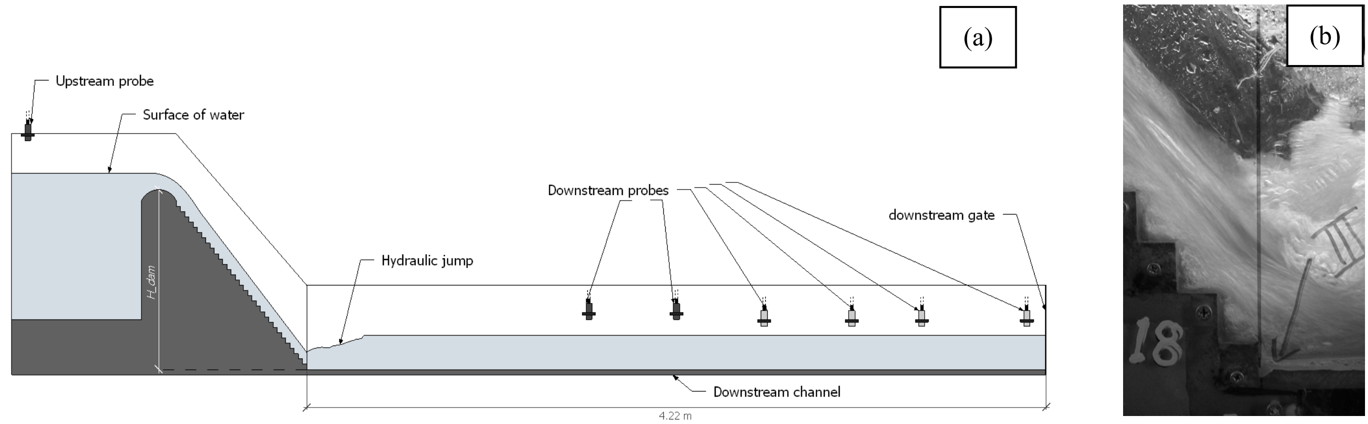

Figure 4: Sketch of the experimental facility (a) and localization of the hydraulic jump at the spillway toe (b).

The water depth at the spillway toe is thus equal to the first conjugate depth h_1 of the jump. To solve the conjugate depths equation (equation (4)), the second conjugate depth h_2 has been evaluated as the mean value measured using a maximum of six ultrasonic probes arranged along the downstream part of the channel. Indeed, friction effects along the subcritical section of the channel have been found to be negligible.

$$h_1 = h_2 \frac{\sqrt{1+8\left(\frac{h_c}{h_2}\right)^3}-1}{2} . \tag{4}$$

The residual energy E_p in the horizontal channel at the spillway toe can be calculated from the first conjugate depth h_1 using equation (5), similar to equation (3).

$$E_p = h_1 + \frac{(q/h_1)^2}{2g} . \tag{5}$$

To evaluate the residual energy at the spillway toe in the spillway, the local head loss due to the slope change at the horizontal channel inlet needs to be considered. To evaluate this local head loss, only the results of the tests carried out with the ogee crested weir and with an H_{dam}/h_c ratio higher than the limit provided by equation (1) have been considered. In practice, working with 90% of the limit given by equation (1) and with our geometry of facility, we had $H_{dam,u}/h_c$ equal to 18.4. For these tests, the flow in the spillway is uniform and the corresponding residual energy $E_{f,u}$ may be computed using equation (2). ΔE_{impact} may thus be simply computed as

$$\Delta E_{impact} = E_p - E_{f,u} . \tag{6}$$

From ΔE_{impact} values and assuming a classical form for the local head loss equation, we obtain

$$\Delta E_{impact} = k \frac{(q/h_1)^2}{2g} \tag{7}$$

where h_1 is the first conjugate depth giving by equation (4), a constant local head loss coefficient k equal to 0.627 is found (Figure 5).

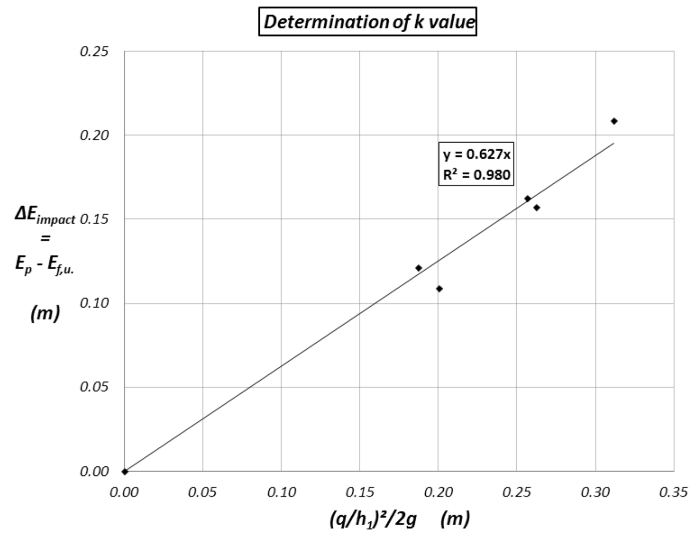

Figure 5: Determination of the local head loss coefficient k value at the spillway toe.

Whatever the discharge, the theoretical residual energy head in uniform flow conditions at the spillway toe in the horizontal channel E_p^{uni} may be calculated as

$$E_p^{uni} = E_{\max} \frac{F}{\dfrac{H_{dam}}{h_c} + \dfrac{3}{2}} - k \frac{(q/h_1)^2}{2g}. \tag{8}$$

E_p^{uni} is only a function of the spillway characteristics and discharge, and not of the upstream weir type. The comparison of this theoretical value with the residual energy measured at the spillway toe is shown in Figure 6 for all the PKW geometries and discharges considered in this study.

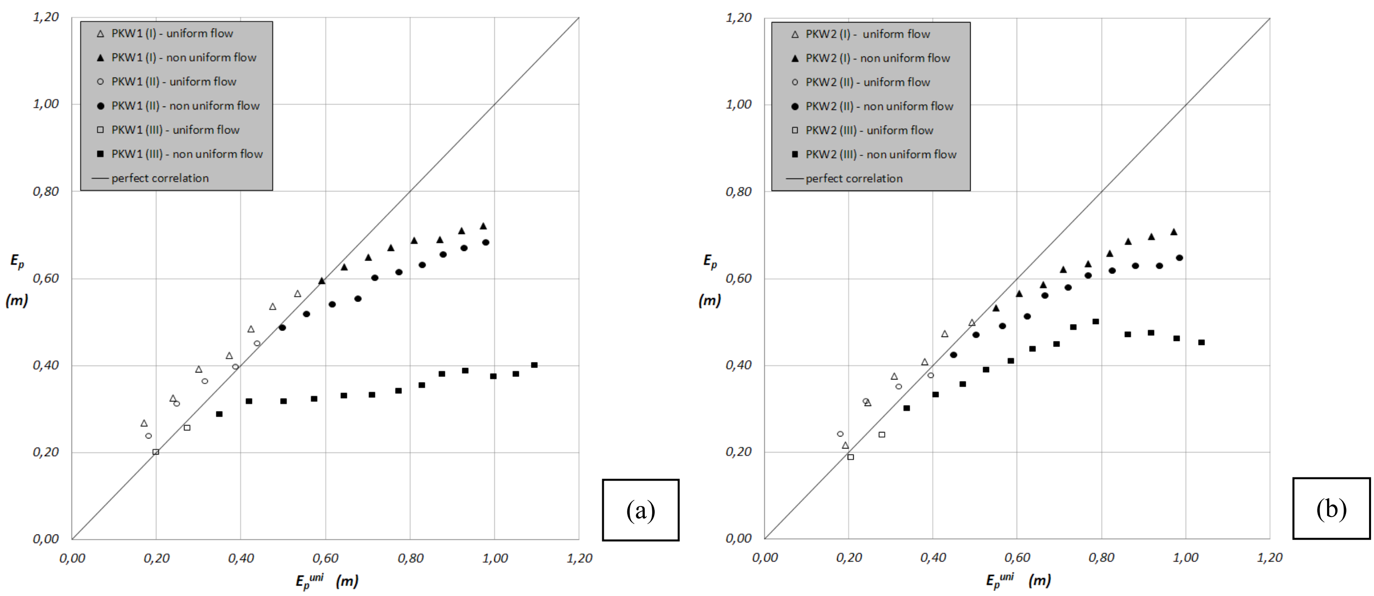

Figure 6: Correlation between the theoretical uniform energy at the spillway toe E_p^{uni} and the measured residual energy E_p for PKW configurations. White symbols for uniform flow energy and black symbols for non-uniform flow energy.

Figure 7 shows the ratio between E_p^{uni} and E_p as a function of $H_{dam,u}/h_c$. The critical dam height to reach uniform flow conditions downstream of the ogee-crested weir as defined by Boes and Hager (2003a) appears too high when the spillway is fed by means of a PKW. The limit ratio for stepped spillway fed by PKW is around 14.5. This value is valid for the tested spillway and weir geometry. Nevertheless, these results prove that using a PKW as upstream weir with a stepped spillway enables to reach uniform flow conditions on a shorter spillway compared to the length needed downstream of a standard ogee crested weir.

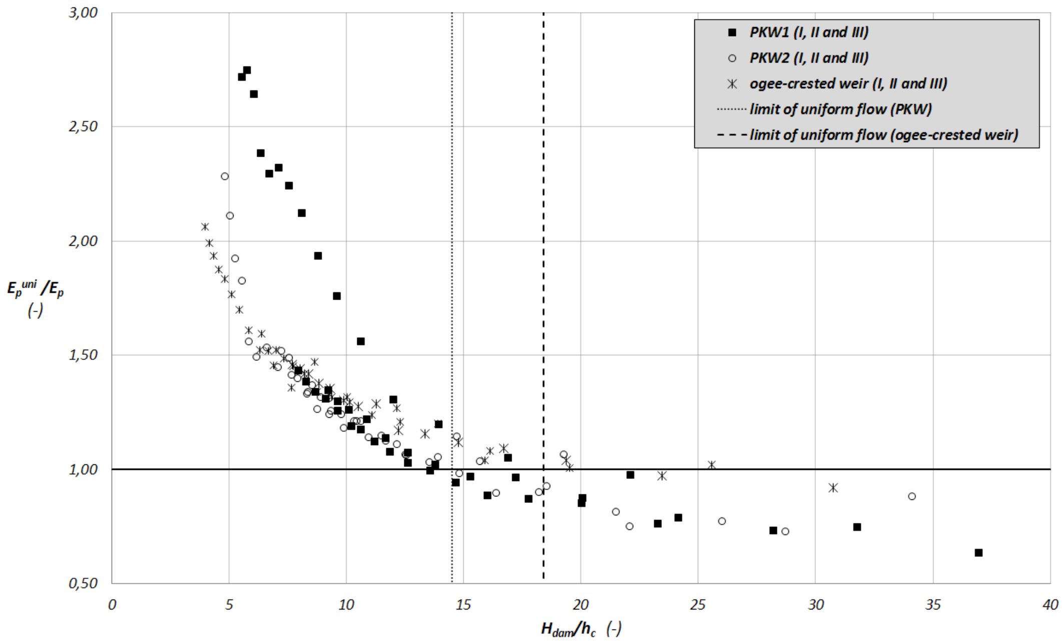

Figure 7: Ratio between theoretical uniform flow energy E_p^{uni} and measured residual energy E_p at the spillway toe as a function of non-dimensional dam height H_{dam}/h_c.

3.3 *Scale effects*

Scale effects have been analysed to evaluate whether the conclusions of the present study, resulting from scale model tests, can be upscaled to a prototype scale. Indeed, highly turbulent air-water flows cannot be modelled without scale effects when using Froude similitude, because of the significant role of viscosity and surface tensions (Boes and Hager, 2003b). For a true similarity of the aeration process between model and prototype, the Froude, the Reynolds and the Weber similarity laws should be fulfilled simultaneously (Kobus, 1984). Therefore, care must be taken when upscaling model results to the prototype scale.

Kobus (1984) proposes Reynolds numbers *Re* with flow depth as reference length (equation (9)) equal or higher to 10^5 to minimize viscous effects.

$$\mathrm{Re} = \frac{q}{\nu} > 10^5 . \tag{9}$$

Rutschmann (1988) and Speerli (1999) concluded both that the Weber number *W*, with the flow depth as the reference length, should be at least 110 for surface tension effect to be negligible (where depth-averaged mixture velocity $\bar{u}_m$ is calculated with the critical depth h_c of the flow on the spillway and L_s is the distance between two successive edges step) – equation (10).

$$W = \frac{\overline{u}_m^{\,2}}{\dfrac{\sigma}{\rho L_s}} = \frac{\left(\dfrac{q}{h_c}\right)^2}{\dfrac{\sigma}{\rho L_s}} \geq 110 . \tag{10}$$

Considering our geometry and the range of tested discharges, Reynolds number varies between 2.4 10^4 and 1.6 10^5 and Weber number varies between 179 and 716.2. Consequently, the results of the study can be extrapolated to a prototype scale without modification.

4 CONCLUSIONS

The present experimental research has been carried at the Laboratory of Engineering Hydraulics, University of Liège, with the main purpose to study the energy dissipation occurring in skimming flow conditions on a stepped spillway downstream of a piano key weir (PKW) by comparison with the theory for stepped spillway downstream of a standard ogee crested weir. The results confirmed the rapid aeration of the flow downstream of piano key weirs due to the specific geometric features of the weir, creating several interacting flows and jets downstream of the structure. Indeed, with PKW geometry, the flow is fully aerated directly after the weir toe, while it is not the case with the ogee-crested weir. The inception point downstream of a PKW is significantly closer to the spillway crest than downstream of an ogee-crested weir.

The results showed that using a PKW as upstream weir enables to reach uniform flow conditions on a shorter stepped spillway compared to the length needed downstream of a standard ogee crested weir. The critical dam height to reach uniform flow condition downstream of the ogee-crested weir appears too high when the spillway is fed by a PKW. Indeed, with our geometry, the limit ratio $H_{dam,u}/h_c$, as defined by Boes and Hager (2003a), for stepped spillway fed by PKW is estimated around 14.5 whereas it is 18.4 downstream of the ogee-crested weir.

Finally, from criteria defined by Kobus (1984), Rutschmann (1988) and Speerli (1999) and having a Reynolds number which varies between 2.4 10^4 and 1.6 10^5, respectively Weber number, between 179 and 716.2, it can be concluded that scale effects have a limited influence in the tested configurations. Consequently, though not experimentally verified, scale effects are not anticipated.

The present experimental research has been carried on with goal to study the energy dissipation along a stepped spillway downstream of a PKW when the non-uniform flow conditions are installed.

NOTATIONS

g	gravitational acceleration
h_0	water depth in the upstream reservoir
h_1	first conjugate depth (supercritical)
h_2	second conjugate depth (subcritical)
h_c	critical depth (on the spillway)
$h_{w,u}$	uniform equivalent clear water depth
k	local loss coefficient
q	specific discharge
s	step height (stepped spillway)
v	flow velocity (hypothesis of uniform distribution on the section)
z	vertical distance between the floor of downstream horizontal channel and the bottom of the upstream reservoir
B	upstream-downstream length of the PKW $B = B_b + B_i + B_0$
B_0	upstream (outlet key) overhang crest length
B_i	downstream (inlet key) overhang crest length
B_b	base length
E_0	energy in the upstream reservoir
$E_{f,u}$	residual energy head at spillway end (uniform flow conditions established)
E_{max}	maximum reservoir energy head
E_p	energy at the spillway toe
F_*	roughness Froude number
H_{dam}	height of dam
$H_{dam,u}$	vertical distance from spillway crest to close uniform equivalent clear water flow
L_u	developed length of the PKW unit along the overflowing crest axis
L_s	$= s/\sin\phi$ distance between step edges, roughness spacing
P	height of the PKW
Re	Reynolds number
T_s	sidewall thickness
W	Weber number
W_u	width of a PKW unit
W_i	inlet key width (sidewall to sidewall)
W_0	outlet key width (sidewall to sidewall)
ΔE_{impact}	local head loss due to flow impact at the change of slope between spillway and downstream horizontal channel
ϕ	slope of stepped spillway
ν	kinematic viscosity of water
ρ	density of water
σ	surface tension between air and water

REFERENCES

Boes R.M. & Hager W.H. (2003a). Hydraulic design of stepped spillways. Journal of hydraulic engineering ASCE., Vol. 129, N°9, 671-679.

Boes R.M. & Hager W.H. (2003b). Two-Phase Flow Characteristics of Stepped Spillways. Journal of hydraulic engineering ASCE., Vol. 129, N°9, 661-670.

Chanson H. (1994). Hydraulics of skimming flows over stepped channels and spillways. Journal of Hydraulic Research, Vol. 32, N°3, 445-460.

Dewals B.J., André S., Schleiss A. & Pirotton M. (2004). Validation of a quasi-2D model for aerated flows over mild and steep stepped spillways. In Liong, Phoon & Babovic (eds). Proc. Of the 6th Int. Conf. on Hydroinformatics, Singapore. Vol. I, 63-70. World Scientific Publishing Company.

Erpicum S., Machiels O., Archambeau P., Dewals B. & Pirotton M. (2011). Energy dissipation on a stepped spillway downstream of a Piano Key Weir – Experimental study. Labyrinth and Piano Key Weirs-PKW 2011. Erpicum et al. (eds). Taylor & Francis Group, London, 105-111.

Hager W.H. & Boes R.M. (2000). Backwater and drawdown curves in stepped spillways flow. Proc. Int. Workshop on Hydraulics of Stepped Spillways, VAW, ETH Zurich, H.-E. Minor and W.H. Hager, eds., Balkema, Rotterdam, The Netherlands, 129-136.

Ho Ta Khanh M., Truong Chi Hien & Nguyen Thanh Hai. (2011). Main results of the PK Weir model tests in Vietnam (2004 to 2010). International Workshop on Labyrinth and Piano Key Weirs, Liège, Belgium..

Kobus H. (1984). Local air entrainment and detrainment. Proc. IAHR Symp. on Scale Effects in Modeling Hydraulic Structures (H. Kobus, ed.) Esslingen, Germany, 4.10: 1-10.

Matos J. & Quintela A. (1994). Discussion of jet flow on stepped weirs. Journal of Hydraulic Engineering, ASCE, Vol. 120, 443-444.

Paternoster C. (1963). Mécanique des fluides, La cinématique des fluides (Tôme 2). Editions Vyncke.

Rutschmann P. (1988). Belüftungseinbauten in Schussrinnen (Aerator devices in spillways). Doctoral Dissertation. Mitteilung N° 97, Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie, ETH Zürich (in German).

Shvainshtein A.M. (1999). Stepped spillways and energy dissipation. Hydrotechnical construction. N°33(5), 275-282.

Silvestri A. (2012). Étude de la dissipation d'énergie sur un coursier en marches d'escalier en aval d'un évacuateur de crue de type PKW. Mémoire de fin d'études. Faculté des sciences appliquées, Université de Liège (URL : http://orbi.ulg.ac.be/retrieve/145402/198658/rapport%20TFE%20v.FINALE.v1.pdf).

Speerli J. (1999). Strömungsprozesse in Grundablasstollen (Flow phenomena in bottom outlets). Doctoral Dissertation. Mitteilung N° 163, Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie, ETH Zürich (in German).